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The Impact of a Cabri Learning Environment on Students' Level of Reasoning

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Abstract: The study aimed at investigating the impact of a Cabri learning environment on the level of reasoning in geometry and whether this impact affects differentially grade eight students of different levels of math achievement. The teaching experiment involved an experimental group (Cabri environment) and a control group (Cabri–free) environment. The Structure of Learned Outcomes (SOLO) taxonomy was used to assess the reasoning level in the student–produced proofs of the posttest. Two scores were generated from the posttest scores: A SOLO score and a SOLO level score. The results indicated that: (a) there was a significant difference between the control and experimental group on the SOLO score in favor of the experimental group and this difference was more pronounced for the low achievers than high and average achievers; (b) the percentage of students at the pre–structural SOLO level was less for the experimental group than that of the control group whereas the percentage students at the multi–structural SOLO level was higher for the experimental group than that in the control group. The findings were interpreted theoretically and empirically.

Key words: Cabri; Computer learning environment; Teaching experiment; Reasoning

Introduction

Cabri geometry is a medium in which students can explore, experiment, and manipulate geometric diagrams. One important feature of Cabri is the dragging test which allows the user to investigate invariant elements when dragging the figure.

Mathematics educators have targeted the development of deductive reasoning as one major goal of mathematics education and considered proof production as a tool in developing deductive reasoning. However, middle graders find difficulty in developing proofs since they often do not see any reason to develop proofs (Hadas, Hershkowitz, & Schwarz, 2000). On the other hand, claims have been made regarding the role of technology in promoting different aspects of mathematics learning. Cabri with its features of visualization, experimentation, and instant feedback attracted researchers to study its integration in teaching and learning geometry.

A number of researchers focused on investigating the *mediating role* of Cabri in promoting students' understanding of the nature of the mathematical proof and in enhancing their proof skills. These studies indicated that the Cabri environment helped students' justifications progress from empirical towards deductive justifications (Healy & Hoyles, 2001; Jones, 2000; Mariotti, 2000, 2001; Marrades & Gutierrez, 2000). On the basis of these studies, it was concluded that Cabri environment helps students connect the empirical and theoretical aspects of geometry. These studies targeted middle and high level achievement groups and were based on the qualitative analysis of observations, interviews, field notes, and student computer files.

Other researchers investigated the impact of the *inspirational role* of Cabri in helping students to develop an understanding of the need for abstract justifications and proofs (Hadas, Hershkowitz, & Schwarz, 2000; Hölzl, 2001; Lopez–Real & Leung, 2002). The inspirational role of Cabri reflects itself in situations where students construct their figures, obtain a situation contradictory to their hypothesis, test the validity of their constructions under dragging, and look for justifications for the constructions. In general these studies supported the hypothesis that Cabri stimulated students' thinking and inspired them to develop justifications.

Other researchers studied the impact of the Geometer's Sketchpad, a software similar to Cabri, on students' level of reasoning. They studied the different roles of the Geometer's Sketchpad in learning geometry (e.g., Almeqdadi, 2000; Choi–Koh, 1999; McClintock, Jiang, & July, 2002; Hannafin, 2004) and showed that Sketchpad had a positive effect on the quality of reasoning in geometry.

The purpose of the present study was to investigate the impact of a Cabri learning environment on the level of reasoning in geometry and whether this impact affects differentially grade eight students of different levels of math achievement. The use of Cabri for enhancing students' skills of proving and justification has been a controversial educational issue. A number of researchers believed that the use of Cabri would impede the development of formal proofs since students make use of exhaustive checking on the screen and they become convinced of the validity of their conjectures (Hanna, 2000). The present study differs in many ways from other studies which investigated the mediating role of Cabri in bridging the gap between the empirical and theoretical aspects of a proof (Mariotti, 2000). First, this study utilized the achievement level of the student as a moderator variable to study the differential effect of a Cabri learning environment on the level of reasoning, whereas most studies targeted above and high achievers. Second, studies using Cabri employed descriptive qualitative analysis of observations, interviews, field notes, and computer students' files, whereas, the present study involved a teaching experiment in a natural classroom setting where a mandated curriculum was used. Third, while previous studies using Cabri targeted the improvement of students' understanding of proof, the present study investigated the impact of a Cabri learning environment on the level of reasoning in geometry.

Method

Participants

The sample consisted of grade eight Lebanese students who were studying geometry according to the Lebanese curriculum which requires students in grades seven, eight, and nine to develop proofs in plane geometry. The study was conducted in a private school in Beirut, whose students mostly come from middle socio–economic background. A sample of 42 grade eight students participated in the study. In this school, the administration assigns students to sections, at the beginning of the year, randomly but stratified according to their achievement level. Two sections out of three were chosen randomly to participate in the study. Each section had 21 students: (a) 20% high achievers; (b) 60% average students; and (c) 20 % low achievers.

One section was the experimental group which used Cabri in learning geometry while the other was the control group which learned the same topics without using Cabri. The dependent variable was the level of reasoning in proofs produced by students, assessed by the *Structure of Learned Outcomes* (SOLO) taxonomy (Jurdak, 1991). The moderator variable was the student's achievement level that has three levels: high (general average of the pervious academic year in math above 80), average (between 60 and 79), and low (below 60).

Instruments

An achievement test that was constructed by the researchers included three geometry problems which involve constructing proofs that were derived from previously learned properties. For instance, problem 1 and problem 3 each included three items which require the use of the midpoint theorem, the definition and the properties of the parallelogram in writing proofs. While problem 2 included two items in which the student has to develop a proof using the midpoint theorem and the properties of the isosceles trapezoid. The content validity of the test was confirmed by the school mathematics coordinator. A pilot study was performed on a sample of 25 students and item analysis indicated a Cronbach alpha of 0.75.
The achievement test was scored by using a rubric based on SOLO taxonomy that defines five scores ranging from 0 (restating the given), 1 (responses based on drawing or measuring), 2 (responses uses logical reasoning with no justifications), 3(responses present reasonable justifications deduced from the figure or the given),

to 4 (logical present justified arguments that integrates previously learned theory and abstract principles to the given in the problem). A sample of 10 papers was scored independently by the two researchers using the SOLO taxonomy and the degree of agreement was 81%.

Design and procedures

The study followed the posttest control group design. This design controls internal validity threats and some of the external validity threats of the experimental work. The combination of the assumed random assignment of students to experimental and control groups and the presence of a control group provide control for most sources that threaten internal validity.

The experiment was conducted in three phases during the last semester (three months) of the academic year 2006/07. Both groups had learned to develop proofs in grade seven, so they had experienced developing proofs during at least one academic year. Before implementing the experiment, all the students in the experiment did not have any chance to use Cabri in learning geometry, but they were trained to use the software in the first phase of the study. The novelty element of Cabri use was reduced through introducing Cabri to both groups in the first phase which involved six 50–minutes sessions. Different activities were designed for exploring Cabri in which students in the control and experimental group were acquainted with the software before starting the experiment.

The experimental group learned geometry while using Cabri for 12 weeks, for two sessions per week. However, the control group learned the same topics concurrently and for the same period but without using Cabri.

Treatment

The second phase of the study was carried over 12 weeks, the experimental group used Cabri in deducing the properties of the parallelogram and the isosceles trapezoid, proving the midpoint theorem and developing new proofs utilizing individual work instructional format. The control group performed the same activities utilizing individual work instructional format but using the geometric set, paper, and pencil. Figure 1 summarizes phase 2 of the experiment.

In both groups, the teacher led the classroom discussions to develop proofs and justifications and posed questions that engaged the students in high reasoning. The teacher asked different questions while performing the activities. In the experimental group, the teacher posed questions to justify the validity of the students' conjectures by using Cabri. Questions like "What properties of the parallelogram are persevered when dragging a vertex?" "Why do you think your conjecture is true?" "Drag the vertices of the triangle, does the midpoint theorem hold?" "Justify your thinking". The teacher's question directed the students to use Cabri efficiently.

Session	Duration	Activity
Parallelogram		
First session Second session	50 minutes 50 minutes	Exploring the properties of the parallelogram: sides, diagonals and angles
Third session Fourth session	50 minutes 50 minutes	Proving quadrilaterals parallelograms with the definition or using sufficient and necessary properties
Isosceles trapezoid		
First session Second session	50 minutes 50 minutes	Exploring the properties of the isosceles trapezoid: sides, diagonals and angles
Third session Fourth session	50 minutes 50 minutes	Proving a quadrilateral is an isosceles trapezoid with the definition or using sufficient and necessary properties
Midpoint theorem		
First session	50 minutes	Proving the midpoint theorem
Second session Third session Fourth session	50 minutes 50 minutes 50 minutes	Using the midpoint theorem in constructing new proofs

Figure 1. Summary of Phase 2 of the experiment

However, the questions posed in the control group sessions focused at the abstract learned mathematical knowledge. For example, the students were asked to justify their conjectures using learned axioms and properties. They deduced the properties of the parallelogram and the isosceles trapezoid through proving congruent triangle, referring to alternate equal angles and using the properties of isosceles triangles. The students used the learned properties about triangles, angles and congruency of triangles in developing new proofs and justifications. The same teacher, the second author, played the role of a facilitator that helped students in working out the activities, explained some geometric concepts, and provided students with feedback. Both groups did the same activities and they took the same test. A sample lesson from each treatment was audio–taped in order to check on the compliance of the teacher with the requirements of the treatments.

The last phase of the study was the assessment phase in which both groups took a 50–minute test. The experimental group did the test in the lab using Cabri while the control group took the test in their classroom. The experimental group saved their work in the computers while the control group handed in papers.

Data analysis

For each student on each of the eight items of the test assigned a SOLO level according to the SOLO taxonomy. For each student two SOLO measures were generated: (1) the *SOLO score* was defined as the sum of the SOLO levels of the items; and, (2) the *SOLO level* was calculated by finding first the mode of the SOLO levels of the items for each problem and then the mode of the SOLO levels of the three problems. The data obtained were analyzed by appropriate statistical techniques like the ANOVA, *t* test for independent samples and cross tabulation. A qualitative analysis was done on a sample of the computer files of students in the Cabri group in order to understand the mediation role of Cabri in the development of students' thinking.

Results

The study compared the level of reasoning in the proofs produced by two–grade eight groups–one instructed in a Cabri–free learning environment and the other group instructed in a Cabri–based learning environment. Table 1 shows the SOLO score mean and standard deviation of the control and experimental groups by achievement level. The calculated *t* value for the mean difference between the control and the experimental groups for each of the high and average student achievement levels was not statistically significant ($p>0.1$), but was significant for the low achievement level ($p<0.1$)

Table 2 presents the analysis of variance (ANOVA) using group as the independent variable, the achievement level as a moderator variable, and the SOLO score as a dependent variable. The ANOVA results indicated that there was a significant difference between the control and experimental groups ($p< 0.1$) in favor of the latter and a significant difference between the achievement levels ($p< 0.01$) (the higher the achievement level the higher the SOLO score). However, there was no significant interaction between the treatment and the achievement level.

Table 1
Mean and Standard Deviation of the SOLO Score of the Control and Experimental Groups by Achievement Level

Achievement level	Group					
	Control			Experimental		
	N	M	(SD)	N	M	(SD)
High	4	17.75	(2.63)	4	20.00	(3.16)
Average	12	15.33	(5.96)	12	16.92	(3.50)
Low	5	8.80	(3.49)	5	12.40	(2.19)
All	21	14.24	(5.79)	21	16.43	(3.99)

Table 2
Group by Achievement Level ANOVA with SOLO Score as Dependent Variable

Source	*SS*	*df*	*MS*	*F*	*p*
Group	51.80	1	51.80	2.89*	.098
Achievement level	339.43	2	169.71	9.48**	.000
Group × Achievement level	7.18	2	3.59	.20	.819
Error	644.33	36	17.90		
Total	10916.00	42			

**p<0.01, *p<0.1

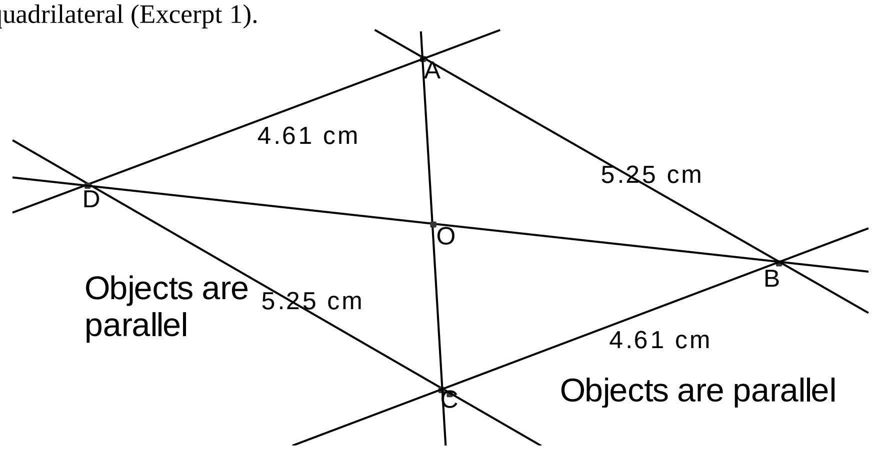
Cross tabulation of the group by the student SOLO level indicated that the percentage of students in the experimental group compared to the control group was less in the pre–structural level (9.5 % vs. 33.3%) but higher in the multi–structural level (57.1 % vs. 38.1%). Moreover, the absolute value of the adjusted standardized residual which indicates the contribution of the cell was highest for the pre–structural level (1.9) followed by the multi–structural level (1.2) (see Table 3). This suggests that the experimental group, compared to the control group, had on the average, a higher SOLO level.

Table 3
Cross Tabulation of Group by Student SOLO Level

SOLO Level	Group					
	Control			Experimental		
	N	% within group	Adjusted Residual	N	% within group	Adjusted Residual
Pre–structural	7	33.3	1.9	2	9.5	-1.9
Uni–structural	1	4.8	-1.1	3	14.3	1.1
Multi–structural	8	38.1	-1.2	12	57.1	1.2
Relational	5	23.8	.4	4	19	-.4

The collected Cabri files in this study showed more student involvement of experimental group in the learning experience than the Cabri–free group. The students who used Cabri in performing the parallelogram and the isosceles trapezoid activities measured the distances between points, measured angles of the quadrilaterals, and checked parallel lines. The computer files showed that students used measures of angles and sides and Cabri commands indicating parallel lines (see Excerpt 1 below). While some students in the control group handed in incomplete work since they were unable to engage in proof activities. The students just constructed figures, rewrote the given and stated conclusions that need to be proved. The experimental group students' work included statements justifying the properties but many of these justifications were obtained from measuring and exploring the figures. Some students used measures of segments in writing the proof of the midpoint theorem, while the activities performed by the control group contained justifications based on previously learned axioms and properties. The control group used congruent triangles to prove opposite sides in a parallelogram are equal then used the properties of the parallelogram in proving the midpoint theorem. Their proofs depended completely on deductions derived from proved theorems and established geometric axioms. Although the teacher guided both groups to develop justifications that integrate learned geometric properties, the experimental group used Cabri for exploring and experimenting properties that encouraged them to look for theoretical justifications while the control group developed proofs derived from theoretical constructs.

Students' Cabri files on the set of activities in the parallelogram, the isosceles trapezoid, and midpoint theorem units showed that the students used Cabri features in their explanations and justifications. The constructions of the parallelograms and the isosceles trapezoid which passed the dragging feature of Cabri motivated the students to develop definitions of the two quadrilaterals and examine their properties. For instance, one Cabri file indicated that the obtained quadrilateral is a parallelogram because lines remain parallel when moving a vertex of the quadrilateral (Excerpt 1).



<u>Excerpt 1</u>

Teacher: Describe the construction.

Student: Locate three non–collinear points.

Teacher: How did you locate the points?

Student: Not on the same line.

Student: Label the points A, B, and C. Draw line passing through A and B and another passing through B and C. glue the lines with intersection of objects.

Teacher: How did you complete the quadrilateral?

Student: Use command parallel lines, select line BC and point A then draw a line parallel to BC and passing through A. use the command parallel line select line AB and point C then draw a line passing through C and parallel to AB.

Student: Glue the two lines by intersection of objects command. Label point D.

Teacher: What is the nature of the obtained quadrilateral?

Student: ABCD is a parallelogram.

Teacher: Justify your answer.

Student: Drag point B, A, or C the opposite lines remain parallel. So ABCD is a parallelogram.

Discussion

The results of this study support the hypothesis that a Cabri learning environment improves the quality of reasoning in geometrical proofs of grade eigth students, as evidenced by the higher SOLO level in the experimental group. Specifically, the results of this study support the positive effect of Cabri on students' level of reasoning.

The Cabri environment had a more positive impact for low–achieving students than middle–or high–achieving students. The results of this study extends research on Cabri that used samples consisting of mostly above average and high achievers (Hadas, Hershkowitz, & Schwarz, 2000; Hölzl, 2001; Jones, 2000; Lopez–Real & Leung, 2002; Marrades & Gutierrez, 2000). Some studies reported that average and high achievers did not benefit from Cabri use in learning geometry (Hadas, Hershkowitz, & Schwarz, 2000; Healy & Hoyles, 2001). A similar result was obtained in this study in that high and average achievers who used Cabri in learning geometry and developing proofs did not perform at a higher level of reasoning than students who learned the same topics without Cabri. However, in this study the low achievers who used Cabri in learning geometry and developing proofs showed a better performance than the low achievers who developed proofs without the aid of Cabri.

The nature of Cabri facilitated more student engagement in the learning process compared to learning in no Cabri setting (Hadas, Hershkowitz, & Schwarz, 2000; Hölzl, 2001; Laborde, 2001). Specifically, the qualitative analysis of students' computer files indicates that the mediating role of Cabri (Healy & Hoyles, 2001; Jones, 2000; Mariotti, 2000, 2001) was crucial in improving the students' level of reasoning. The Cabri mediating role reflected itself in directing the students' thinking to formulate conjectures and look for justifications for the constructions that passed the dragging test. The mediating role of Cabri was also realized in students' development of meanings of geometry theorems (Mariotti, 2000, 2001). It seems that Cabri has a semiotic mediation (from the Vygotskian perspective) (Fosnot, 1996; Schifter & Simon, 1991) role in learning geometric theorems. The dragging function of Cabri, which is used to check the validity of a construction, suggests a theoretical property when the figure is not distorted by dragging. In this process, the students internalize the external signs of Cabri and change them to internal psychological signs which direct their thinking in developing conjectures and proofs.

In conclusion, this study supports the use of a Cabri environment to enhance the level of reasoning in geometry. It further illuminates the role of Cabri features in engaging students in learning as well as mediating the transition from the empirical to the theoretical in mathematical geometrical proofs. These conclusions are constrained by two limitations of this study: The small sample used in the study and the fact that the experiment was limited to grade eight students.

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